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REANALYSIS OF THE MASS DIFFERENCE OF $B_d^0 - \overline{B}_d^0$ WITHIN THE MINIMAL SUPERSYMMETRIC STANDARD MODEL

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ABSTRACT

We present a detailed and complete calculation of the loop corrections to the mass difference $\Delta m_{B_d^0}/m_{B_d^0}$. We include charginos and scalar up quarks as well as gluinos and scalar down quarks on the relevant loop diagrams. We include the mixings of the charginos and of the scalar partners of the left and right handed quarks. We find that the gluino contribution to this quantity is important with respect to the chargino contribution only in a small part of phase space: mainly when the gluino mass is small (~ 100 GeV) and the symmetry-breaking parameter m_S is below 300 GeV. This contribution is also important for very large values of $\tan\beta$ (~ 50) irrespective of the other parameters. Otherwise, the chargino contribution dominates vastly and can be roughly as large as that of the Standard Model. We also present the contribution of the charged Higgs to the mass difference $\Delta m_{B_d^0}/m_{B_d^0}$ in the case $m_b \tan\beta \ll m_t \cot\beta$. This last contribution can be larger than the Standard Model contribution for small values of the Higgs mass and small values of $\tan\beta$.

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I. INTRODUCTION

The mass difference $\Delta m_{B_d^0}/m_{B_d^0} \approx 6.4 \times 10^{-14}$ GeV [1] is an experimental well known value. It was calculated within the standard model, where W bosons and up quarks run in the relevant box diagrams, a long time ago [2–5 and references therein]. These authors pointed out the importance of the top quark mass in the B_d^0 system, which is due to the parameters K_{ij} of the Kobayshi–Maskawa matrix (in the K^0 system the top quark contribution is smaller than the charm quark contribution since $(K_{21}^* K_{22})^2 m_c^2 \geq (K_{31}^* K_{32})^2 m_t^2$ whereas in the B_d^0 system the term $(K_{31}^* K_{33})^2 m_t^2$ is dominant). The measured value of $x_{B_d^0} = \Delta m_{B_d^0} \times \tau_{B_d^0} \approx 0.71$ was unexpectedly high. As explanation it was shown that the top quark mass had to be larger than 45 GeV, which was quite surprising at this time. Nowadays from CDF we know that the top quark mass is almost 4 times that much; with a given value of 174 GeV [6].

Because of such a large top quark mass we have to reconsider the influence of one of the most favoured models beyond the SM, its minimal supersymmetric extension (MSSM) [7], to $\Delta m_{B_d^0}$. A rough estimate of the influence of the MSSM to the mass difference of the K^0 system was first done by several authors [8–10 and references therein]. These authors replaced the W bosons and up quarks by their super partners, the charginos and scalar up quarks. Rough estimate it was, because they neglected the mixing of the scalar partners of the left and right handed up quarks or the mixing of the charginos. Neglecting the mixing of the charginos in general lead to wrong statements as was shown in [11] and the importance of the mixing in the scalar top quark sector due to the heavy top quark can be seen in [12–13].

In this paper we present a detailed and complete calculation of the charginos and scalar up quarks contribution to the mass difference of the B_d^0 system. In the calculation we neglect all masses of quarks besides the top quark mass. As explained above our results cannot be used in the K^0 system.

More than ten years ago it was shown that loop diagrams induce flavour changing couplings of the gluinos to the down quarks and their scalar partners [14–15]. Since this coupling is strong its influence was first analysed in the K^0 system [15–17] and later in the B_d^0 system [18–19]. There it was shown that for a small top quark mass of 40 GeV and masses of the gluinos and scalar down quarks at the lower experimental limit of around 60 GeV at this time, the SUSY corrections are of the same order and even higher than the SM result. In this paper we repeat those calculations and give more general results: we include the mixing of the scalar partners of the left and right handed bottom quarks. This might become important if $\tan\beta = v_2/v_1 \gg 1$ where $v_{1,2}$ are the vacuum expectation values (vev's) of the Higgs particles in the MSSM.

In the next section we present the calculation and discuss the results in the third section. We end with the conclusions.

II. SUSY CORRECTIONS TO THE B_d^0 SYSTEM

In the SM the mass difference in the B_d^0 system is obtained by calculating the box diagrams, where W bosons and up quarks are taken within the loop. After summation over all quarks it turns out that the top quark gives the main contribution to $\Delta m_{B_d^0}$. The result is well known and given by [2]:

$$\frac{\Delta m_{B_d^0}}{m_{B_d^0}} = \frac{G_F^2}{6\pi^2} f_B^2 B_B \eta_t m_W^2 (K_{31}^* K_{33})^2 S(x_t) \tag{1}$$
$$S(x_t) = x_t\{\frac{1}{4} + \frac{9}{4}(1-x_t)^{-1} - \frac{3}{2}(1-x_t)^{-2}\} - \frac{3}{2}\frac{{x_t}^3}{(1-x_t)^3}\ln x_t$$

f_B, B_B are the structure constant and the Bag factor obtained by QCD sum rules and η_t a QCD correction factor [2]. For a large top quark mass their values are given by $f_B = 0.180$ GeV, $B_B = 1.17$ and $\eta_t = 0.55$ [3].

To obtain $\Delta m_{B_d^0}$ within the MSSM we have to calculate all those diagrams as shown in Fig.1, where we also include the so called "mass insertion" diagram denoted with $\otimes$ [8]. This notation means that in this diagram $P_R(\not{k}+m)P_R = mP_R$ remains whereas the other one gives $P_R(\not{k}+m)P_L = \not{k}P_L$ ($P_{L,R}$ are the projection operators). In our calculation we take the full set of couplings as given in Fig.22 and Fig.23 of [20][1]. Furthermore as mentioned above we include the mixing of the charginos[2] as well as the mixing of the scalar partners of the left and right handed quarks, that is instead of the current eigenstates $\tilde{q}_{L,R}$ we work with the mass eigenstates

$$\tilde{q}_1 = cos\Theta\tilde{q}_L + \sin\Theta\tilde{q}_R \qquad \tilde{q}_2 = -\sin\Theta\tilde{q}_L + \cos\Theta\tilde{q}_R \tag{2}$$

In the scalar top quark sector we have the following matrix [22]:

$$M_{\tilde{t}}^2 = \begin{pmatrix} m_{\tilde{t}_L}^2 + m_t^2 + 0.35D_Z & -m_t(A_t + \mu\cot\beta) \\ -m_t(A_t + \mu\cot\beta) & m_{\tilde{t}_R}^2 + m_t^2 + 0.15D_Z \end{pmatrix} \tag{3}$$

Here $D_Z = m_Z^2\cos 2\beta$, $0.35 = T_3^t - e_t\sin\Theta_W$ and $0.15 = e_t\sin\Theta_W$. $m_{\tilde{t}_{L,R}}$ are soft SUSY breaking mass terms, A_t the parameter from the trilinear scalar interaction and μ the mixing mass term of the Higgs bosons.

The mass matrix of the scalar top quark is of importance, when we consider the scalar up quarks and charginos within the loop. The mass matrix of the scalar bottom quark would be of importance when we would take neutralinos and scalar down quarks within the loop. Here we have to be more careful since a while ago it was shown in [14–15], that when including loop effects flavour changing couplings of the scalar partner of the left handed down quarks with the gluinos are created, whereas the couplings of the gluinos with the scalar partner of the right handed down

[1] In Fig.22 b)+d) and Fig.23 b)+d) γ_5 has to be replaced by $-\gamma_5$

[2] A more detailed description of the mixing of the charginos and of the scalar top quarks can be found in [21].

quarks remain flavour diagonal, that is only the scalar partners of the left handed down quarks have to be considered in the relevant loop diagrams to $B_d^0-\overline{B}_d^0$ mixing. For the mass matrix of the scalar bottom quark we therefore have to take at 1 loop level:

$$M_{\tilde{b}}^2=\begin{pmatrix} m_{\tilde{b}_L}^2+m_b^2-0.42D_Z-|c|m_t^2 & -m_b(A_b+\mu\tan\beta) \\ -m_b(A_b+\mu\tan\beta) & m_{\tilde{b}_R}^2+m_b^2-0.08D_Z \end{pmatrix} \tag{4}$$

with $T_3^b-e_b\sin\Theta_W=-0.42$ and $e_b\sin\Theta_W=-0.08$. The model dependent parameter c plays a crucial role in the calculation of the gluino and scalar down quark contribution to the mass difference in the B_d^0 system. The value of c is negative and of order 1 ($|c|$ increases with the soft SUSY breaking mass term m_S and decreases with the top quark mass [18]). In the following we take $c=-1$, although we keep in mind that it is more likely smaller in magnitude. The mixing term might only get important in the case $\tan\beta\gg 1$.

When calculating the box diagrams in Fig.1 we used the rules given in appendix D in [7]. After a lengthy but straightforward calculation we obtain the following result when charginos and scalar up quarks are running on the loop:

$$\begin{aligned}
\frac{\Delta m_{B_d^0}}{m_{B_d^0}} =& \frac{G_F^2}{4\pi^2}f_B^2B_Bm_W^4(K_{31}^*K_{33})^2[Z_{11}^{\tilde{W}}-2Z_{31}^{\prime\tilde{W}}+\tilde{Z}_{33}^{\tilde{W}}] \qquad (5)\\
Z_{11}^{\tilde{W}} =& \sum_{i,j=1,2} V_{i1}^2V_{j1}^2G_{\tilde{u}\tilde{u}}^{ij}\\
Z_{31}^{\prime\tilde{W}} =& \sum_{i,j=1,2} V_{i1}V_{j1}\Big\{V_{i1}V_{j1}[c_{\Theta_t}^2G_{\tilde{t}_1\tilde{u}}^{ij}+s_{\Theta_t}^2G_{\tilde{t}_2\tilde{u}}^{ij}]\\
&+\frac{m_t^2}{2m_W^2\sin^2\beta}V_{i2}V_{j2}[s_{\Theta_t}^2G_{\tilde{t}_1\tilde{u}}^{ij}+c_{\Theta_t}^2G_{\tilde{t}_2\tilde{u}}^{ij}]\Big\}\\
\tilde{Z}_{33}^{\tilde{W}} =& \sum_{i,j=1,2}\Big\{V_{i1}^2V_{j1}^2[c_{\Theta_t}^4G_{\tilde{t}_1\tilde{t}_1}^{ij}+2c_{\Theta_t}^2s_{\Theta_t}^2G_{\tilde{t}_1\tilde{t}_2}^{ij}+s_{\Theta_t}^4G_{\tilde{t}_2\tilde{t}_2}^{ij}]\\
&+\frac{m_t^2}{m_W^2\sin^2\beta}V_{i1}V_{i2}V_{j1}V_{j2}[c_{\Theta_t}^2s_{\Theta_t}^2(G_{\tilde{t}_1\tilde{t}_1}^{ij}+G_{\tilde{t}_2\tilde{t}_2}^{ij})+(c_{\Theta_t}^4+s_{\Theta_t}^4)G_{\tilde{t}_1\tilde{t}_2}^{ij}]\\
&+\frac{m_t^4}{4m_W^4\sin^4\beta}V_{i2}^2V_{j2}^2[s_{\Theta_t}^4G_{\tilde{t}_1\tilde{t}_1}^{ij}+2c_{\Theta_t}^2s_{\Theta_t}^2G_{\tilde{t}_1\tilde{t}_2}^{ij}+c_{\Theta_t}^4G_{\tilde{t}_2\tilde{t}_2}^{ij}]\Big\}\\
G_{ab}^{ij} :=& \tilde{F}_{ab}^{ij}+2\,{}_M\tilde{F}_{ab}^{ij}
\end{aligned}$$

$c_{\Theta_t}^2=\cos^2\Theta_t$, $s_{\Theta_t}^2=\sin^2\Theta_t$ and $\sin\beta$ can be extracted from $\tan\beta$. $\tilde{F}_{ab}^{ij}$ and ${}_M\tilde{F}_{ab}^{ij}$ are given in the appendix A. $m_{i,j}=m_{\tilde{W}_{i,j}}$ are the mass eigenvalues of the charginos and $m_{\tilde{u},\tilde{t}_{1,2}}$ the masses of the scalar up quark and the eigenstates of the scalar top quark including the mixing. V_{ij} and U_{ij} are the diagonalizing matrices of the charginos as given in eq. C19 in [7] taken to be real. $\tilde{F}_{ab}^{ii}$ and ${}_M\tilde{F}_{ab}^{ii}$ are the same functions as given in eq.C.2 in [23].

Similarly, when gluinos and scalar down quarks run on the loop we obtain [3]

$$\begin{aligned}\frac{\Delta m_{B_d^0}}{m_{B_d^0}} =& -\frac{\alpha_s^2}{54} f_B^2 B_B (K_{31}^* K_{33})^2 [Z_{11}^{\tilde g} - 2Z_{31}'^{\tilde g} + \tilde Z_{33}^{\tilde g}] \\ Z_{11}^{\tilde g} =& T_{\tilde d \tilde d}^{\tilde g} \\ Z_{31}'^{\tilde g} =& c_{\Theta_b}^2 T_{\tilde b_1 \tilde d}^{\tilde g} + s_{\Theta_b}^2 T_{\tilde b_2 \tilde d}^{\tilde g} \\ \tilde Z_{33}^{\tilde g} =& c_{\Theta_b}^4 T_{\tilde b_1 \tilde b_1}^{\tilde g} + 2 c_{\Theta_b}^2 s_{\Theta_b}^2 T_{\tilde b_1 \tilde b_2}^{\tilde g} + s_{\Theta_b}^4 T_{\tilde b_2 \tilde b_2}^{\tilde g} \\ T_{ab}^{\tilde g} :=& 11 \tilde F_{ab}^{\tilde g} + 4 {}_M\tilde F_{ab}^{\tilde g}\end{aligned} \tag{6}$$

$\tilde F_{ab}^{\tilde g}$ and ${}_M\tilde F_{ab}^{\tilde g}$ can be obtained by setting $m_i = m_j \leftrightarrow m_{\tilde g}$ in the functions given in appendix A. Since we neglected all quark masses [4] beside the top quark mass we made use of $Z_{11} = Z_{12} = Z_{21} = Z_{22}$ and $Z_{13} = Z_{31} = Z_{32} = Z_{23}$. Since the mixing of the scalar quark is proportional to the quark masses $c_{\Theta_b}^2 = \cos^2\Theta_b \approx 1$, only for large values of $\tan\beta$ the mixing angle of the scalar bottom quark mass becomes more important.

Finally we also want to comment on the charged Higgs boson contribution to the mass difference in the B_d^0 system. In the case of neglecting bottom quark mass that is $m_b \tan\beta \ll m_t \cot\beta$ we obtain [25]:

$$\begin{aligned}\Delta m_{B_d^0}/m_{B_d^0} =& \frac{G_F^2}{16\pi^2} m_t^4 \cot^4\beta f_B^2 B_B (K_{31}^* K_{33})^2 \big\{ \tilde F_{H^+H^+}^{tt} \\ &+ 2\tan^2\beta [\tilde F_{H^+W^+}^{tt} + 4(m_W/m_t)_M^2 \tilde F_{H^+W^+}^{tt}] \big\}\end{aligned} \tag{7}$$

When one has $m_b \tan\beta \sim m_t \cot\beta$ one should not neglect the bottom quark mass when calculating the box diagram; this complicates greatly the calculations.

III. DISCUSSIONS

We now present those contributions for different values of Higgs, gaugino, gluino and scalar quark masses. We also vary *tanβ* and the symmetry-breaking scales. As input parameter we take $m_{\rm top} = 174$ GeV, $m_{\rm b} = 4.5$ GeV, $\alpha = 1/137$ and for the strong coupling constant $\alpha_s = 0.1134$. For a top quark mass of 174 GeV the SM result eq.1 gives a value of 4.67×10^{-16}.

We first show on fig.2 the charged Higgs contribution. We see that for small values of $\tan\beta$ and light Higgs, this contribution can exceed that of the SM. For $\tan\beta = 1$, even for very large Higgs masses, this contribution is still 20% of the

[3] Eq.6 agrees with [18–19] for $c_{\Theta_b} = 1$.

[4] In calculating the box diagram it is safe to neglect the bottom quark mass, since in the coupling there is no $\tan\beta$ dependance and therefore we can use $m_b \ll m_t$.

SM contribution. However, this contribution goes down very quickly when $\tan\beta$ increases. Given our approximation ($m_b \tan\beta \ll m_t \cot\beta$), we cannot exceed $\tan\beta \sim 5$ and still trust our results. Note that it has been shown in [24] [5], that this contribution goes down very quickly when $\tan\beta$ is large; we certainly see this trend.

In figs. 3 and 4, we show the chargino and gluino contributions. The global behaviour is clear: for small gluino mass and small values of m_S, the gluino contribution is important no matter what values the other parameters have. On the other hand, for large gluino mass and/or large values of m_S, the chargino contribution vastly dominates. The only exception to this rule is for very large values of $\tan\beta$ (~ 30 or higher). In this very special case, the gluino contribution can be important, even for large gluino mass and values of m_S up to $400 - 500$ GeV. This is due to the fact that such large values of $\tan\beta$ can push down the mass of one of the scalar b-quark eigenstates; well below the scalar top-quark eigenstates. Even requiring all scalar-quark masses to be larger than the experimental limit of 92 GeV [6] it is still possible for the gluino contribution to be larger than the chargino contribution by a factor of 6 or so at the minium allowed value of m_S. We also note that values of μ like 100 GeV or 400 GeV for a value of m_{g_2} of 200 GeV will increase (in magnitude) the contributions from the charginos while negative values of μ will decrease them in magnitude. Furthermore, the effects of the mixing of the scalar partners with the top and bottom quarks are more important for large values of m_S: the contributions from the charginos don't decrease as quickly with the mixing. For small values of m_S, there is also an enhancement.

The factor c that enters the b-quark mixing matrix is also very important. We find that reducing it from 1 to 1/2 reduces the gluino contributions by a factor of ~ 8 for small values of m_S ($\sim 200\ GeV$) while it reduces them by a factor of ~ 4 for very large values of m_S ($\sim 1\ TeV$). The first factor will vary with m_{g_2}, μ and $tan\beta$ but the reduction of 4 for large m_S is rather stable. Typically it will vary between 3.5 and 4.1. It can be understood by the fact that for those large scales, the mass eigenvalues are almost insensitive to c but the mixing angles are almost linearly dependant on c.

Finally, one must not forget that in eq.(5) and eq.(6) $K^*_{31}K_{33}$ have not necessarily the same values as in the SM. This was shown in eq.(33) in [14]: the Kobayashi–Maskawa matrix in the couplings of the charginos to quarks and scalar quarks is multiplied by another matrix V_u, which can be parametrized as follows:

$$V_u = \begin{pmatrix} 1 & \varepsilon_u & \varepsilon_u^2 \\ -\varepsilon_u & 1 & \varepsilon_u \\ -\varepsilon_u^2 & -\varepsilon_u & 1 \end{pmatrix} \tag{8}$$

so that $K \equiv V_u \cdot K_{SM}$. Clearly, if $\epsilon \ll 1$ then $K \sim K_{SM}$. However, with $\varepsilon_u = 0.5$ $K^*_{31}K_{33}$ is enhanced by a factor of 14 over the SM value. This enhancement falls

[5] the authors included the bottom quark mass only for the diagram with two charged Higgs bosons within the loop

[6] this lower limit is model dependant, ref.[1]

quickly as ε_u decreases: to 3 for $\varepsilon_u = 0.3$ and to -0.2 for $\varepsilon_u = 0.1$. Note that because of the uncertainties in K_{SM} this last factor has a large error.

We have a similar matrix in the gluino–down quark–scalar down quark couplings as was shown in eq.(29) in [14]. Here $K = V_d$ where V_d is a similar matrix as V_u in eq.(8). For $\varepsilon_d = 0.1$ $K^*_{31}K_{33}$ is identical to the SM values, whereas $\varepsilon_d = 0.5$ enhances it by 25 and $\varepsilon = 0.3$ by 9. Considering that these values are to be squared in the mass difference of the B^0_D system we can use that enhancement to put limits on $\varepsilon_{u,d}$. In the case at hand, ε_u has to be smaller than 0.2 and ε_d smaller than 0.1 to keep the results lower than the measured value of $\Delta m_{B^0_d}/m_{B^0_d}$. This is not very constraining yet but it is already better than the limit one can get from current data on rare Kaon decays [21].

IV. CONCLUSIONS

In this paper we presented the contributions from charginos and scalar up quarks as well as gluinos and scalar down quarks to the mass difference in the B^0_d system via box diagrams. We gave exact results and included the mixing of the charginos and the mixing of the scalar top and bottom quarks. We have shown that in the case of charginos and scalar top quark the mixing becomes important and leads to an enhancement of the results. Whereas in the case of gluinos and scalar bottom quark its mixing is, as expected, less important, even for higher values of $\tan\beta$ the results are dimisnished only by a few per cents. We have shown that for reasonable values of the SUSY parameters the contribution of the box diagrams with charginos and scalar up quarks can be of the same order as those of the SM diagrams, but with opposite sign. The same goes for the contribution of the gluino and scalar down quarks box diagrams, which has the same sign as the SM contribution. Since we have shown that despite the smallness of the weak coupling constant compared to the strong coupling constant charginos and scalar up quarks cannot be neglected, we believe that for a complete analysis it will be necessary also to include the diagrams with neutralinos and scalar down quarks within the loops [26]; this is also supported by the fact that the lower mass bound of the smallest neutralino by LEP data is only 30 GeV. The contribution from the charged Higgs boson to the mass difference in the B^0_d system can be very important for small values of $tan\beta$ and small Higgs masses. When the Higgs mass becomes large (~ 500 GeV) and/or $2 \leq tan\beta$, this contribution becomes small and even negligible compared to the chargino contribution. We presented the results in the case where $m_b \tan\beta \ll m_t \cot\beta$. An exact calculation including the bottom quark mass therefore is highly desirable.

A complete analysis within the MSSM where all its particles are taken within the relevant box diagram without neglecting any mixing angles and mass eigenvalues might therefore be of special interest and give new bounds on SUSY parameters by the measured value of $\Delta m_{B^0_d}/m_{B^0_d}$. Such a study becomes quite relevant in the light of the upcoming *B-factories*.

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VI. APPENDIX A

For the box diagram we have to calculate the following integrals:

$$F^{\mu\nu}_{abij} := \int \frac{d^4k}{(2\pi)^4} \frac{k^\mu k^\nu}{(k^2-m_a^2)(k^2-m_b^2)(k^2-m_i^2)(k^2-m_j^2)} \tag{A0}$$

$$F^{\mu\nu}_{abij} =: \frac{+ig^{\mu\nu}}{4(4\pi)^2}\tilde{F}^{ij}_{ab}$$

$m_i^2 \neq m_j^2 \neq m_a^2 \neq m_b^2$

$$\tilde{F}^{ij}_{ab} = -\frac{1}{(m_j^2-m_i^2)(m_b^2-m_a^2)}\Big\{\frac{1}{(m_i^2-m_a^2)(m_i^2-m_b^2)}[m_i^4(m_b^2\ln\frac{m_b^2}{m_i^2} - m_a^2\ln\frac{m_a^2}{m_i^2}) - m_i^2 m_a^2 m_b^2 \ln\frac{m_b^2}{m_a^2}] - (m_i^2 \leftrightarrow m_j^2)\Big\} \tag{A1}$$

$m_j^2 \neq m_i^2 \neq m_a^2 = m_b^2$

$$\tilde{F}^{ij}_{aa} = -\frac{1}{(m_j^2-m_i^2)}\Big\{\frac{1}{(m_i^2-m_a^2)}[m_a^2 - \frac{m_i^4}{(m_i^2-m_a^2)}\ln\frac{m_i^2}{m_a^2}] - (m_i^2 \leftrightarrow m_j^2)\Big\} \tag{A2}$$

$m_i^2 = m_j^2 \neq m_a^2 \neq m_b^2$

$$\tilde{F}^{ii}_{ab} = \tilde{F}^{ij}_{aa}(m_a^2 \leftrightarrow m_i^2,\ m_b^2 \leftrightarrow m_j^2) \tag{A3}$$

$m_j^2 = m_i^2 \neq m_a^2 = m_b^2$

$$\tilde{F}^{ii}_{aa} = -\frac{(m_i^2+m_a^2)}{(m_i^2-m_a^2)^2}\{1 - \frac{2m_i^2 m_a^2}{(m_i^4-m_a^4)}\ln\frac{m_i^2}{m_a^2}\} \tag{A4}$$

The second integral is given by:

$$_M F^{ij}_{ab} := \int \frac{d^4k}{(2\pi)^4} \frac{m_i m_j}{(k^2-m_a^2)(k^2-m_b^2)(k^2-m_i^2)(k^2-m_j^2)} \tag{A5}$$

$$_M F^{ij}_{ab} =: \frac{-ig^{\mu\nu}}{(4\pi)^2}\,{}_M\tilde{F}^{ij}_{ab}$$

$m_i^2 \neq m_j^2 \neq m_a^2 \neq m_b^2$

$$_M\tilde{F}^{ij}_{ab} = -\frac{m_i m_j}{(m_j^2-m_i^2)(m_b^2-m_a^2)}\Big\{\frac{1}{(m_i^2-m_a^2)(m_i^2-m_b^2)}[m_i^2(m_b^2\ln\frac{m_b^2}{m_i^2} - m_a^2\ln\frac{m_a^2}{m_i^2}) - m_a^2 m_b^2 \ln\frac{m_b^2}{m_a^2}] - (m_i^2 \leftrightarrow m_j^2)\Big\} \tag{A6}$$

$m_j^2 \neq m_i^2 \neq m_a^2 = m_b^2$

$$_M\tilde{F}^{ij}_{aa} = -\frac{m_i m_j}{(m_j^2 - m_i^2)}\{\frac{1}{(m_i^2 - m_a^2)}[1 - \frac{m_i^2}{(m_i^2 - m_a^2)} \ln \frac{m_i^2}{m_a^2}] - (m_i^2 \leftrightarrow m_j^2)\} \quad (A7)$$

$m_i^2 = m_j^2 \neq m_a^2 \neq m_b^2$

$$_M\tilde{F}^{ii}_{ab} = {}_M\tilde{F}^{ij}_{aa}(m_a^2 \leftrightarrow m_i^2,\ m_b^2 \leftrightarrow m_j^2,\ m_i m_j \rightarrow m_i^2) \quad (A8)$$

$m_j^2 = m_i^2 \neq m_a^2 = m_b^2$

$$_M\tilde{F}^{ii}_{aa} = -\frac{m_i^2}{(m_i^2 - m_a^2)^2}\{2 + \frac{(m_i^2 + m_a^2)}{(m_i^2 - m_a^2)} \ln \frac{m_a^2}{m_i^2}\} \quad (A9)$$

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within an overall sign. This sign is numerically important for small Higgs mass and $tan\beta \sim 1$; for large Higgs mass ($\sim$ 300 GeV or more) or $2 \leq tan\beta$ the difference is not very large. There is also an overall 2/3 which comes from a different definition of B_B.

[26] G. Couture and H. König work in progress.

FIGURE CAPTIONS

Fig.1 The box diagrams with scalar up (down) quarks and charginos (gluinos) within the loop including the mass insertion diagram. Note that in the case of the gluinos its arrow is the other way round.

Fig.2 The ratio of the total amplitude $\Delta m^{\mathrm{H}^+}_{B^0_d}/\Delta m^{\mathrm{SM}}_{B^0_d}$ as a function of M_{H^+} for $tan\beta = 1$ (solid); $tan\beta = 2$ (dash); $tan\beta = 5$ (dash-dot).

Fig.3 The ratios $\Delta m^{\mathrm{Chargino}}_{B^0_d}/\Delta m^{\mathrm{SM}}_{B^0_d}$ and $\Delta m^{\mathrm{Gluino}}_{B^0_d}/\Delta m^{\mathrm{SM}}_{B^0_d}$ as a function of the scalar mass m_S for $tan\beta = 1$ (solid); $tan\beta = 2$ (dash); $tan\beta = 5$ (dash-dot); $tan\beta = 20$ (dot). The negative values for large m_S are the chargino contributions; those of large amplitudes for small m_S are the gluino contributions with $m_{\tilde{g}} = 100\ GeV$; those of small amplitudes for small m_S are the gluino contributions with $m_{\tilde{g}} = 200\ GeV$.

Fig.4 The same as fig. 3 for different values of m_{g_2} and μ. The curves of large amplitudes are the chargino contributions; those of smaller amplitudes are the gluino contributions with $m_{\tilde{g}} = 100\ GeV$; those of smallest amplitudes are the gluino contributions with $m_{\tilde{g}} = 200\ GeV$.